Pacing: Weeks 23 - 30

Interpreting Functions

A. Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by \( f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) \) for \( n \geq 1 \).

B. Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function \( h(n) \) gives the number of person-hours it takes to assemble \( n \) engines in a factory, then the positive integers would be an appropriate domain for the function.★

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

C. Analyze functions using different representations.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

   b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as \( y = (1.02)^t \), \( y = (0.97)^t \), \( y = (1.01)12^t \), \( y = (1.2)^\frac{t}{10} \), and classify them as representing exponential growth or decay.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
## Building Functions

**A. Build a function that models a relationship between two quantities.**

1. Write a function that describes a relationship between two quantities.*
   
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

## Linear, Quadratic, & Exponential Models

**A. Construct and compare linear, quadratic, and exponential models and solve problems**

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

## Interpreting Categorical & Quantitative Data

**B. Interpret expressions for functions in terms of the situation they model**

5. Interpret the parameters in a linear or exponential function in terms of a context.

## Key Student Understandings

**Assessment**

Instructional Guide for Common Core Algebra

- Property of MPS

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Rev July 2016
Common Core Algebra
Critical Area 5: Exponential Relationships and Models

- Students will use recursive processes to construct exponential relationships.
- Students will interpret the key features of exponential functions (constant multiplier, percent change, intercepts, etc.) in multiple contexts.
- Students will create exponential models to represent patterns that arise from real-world contexts.
- Students will compare linear and exponential models, so as to determine which type of model best represents a particular situation.

Formative Assessment Strategies
Evidence for Standards-based Grading
**Common SBG Evidence Items**
Rubric
Investment Assessment (F-BF.A, F-LE.A)
Illegal Fish (F-IF.A, F-LE.A, F-LE.B)
African Black Rhino Population (F-IF.C, S-ID.B, MP4)
Phone Opening Weekend Sales (S-ID.B)
Exponential Growth versus Linear Growth 1 (F-IF.B, F-LE.A, S-IC.A)

Disciplinary Literacy

Disciplinary Literacy Framework
Additional Resources available soon

Common Misconceptions/Challenges
- Arithmetic and geometric sequences do not follow the same formulas.
- Students may attempt to substitute a term number in order to evaluate a recursive formula. Conversely, they may attempt to substitute a prior term in order to evaluate an explicit formula.
- There is sometimes confusion regarding increasing and decreasing behavior. When given a table or graph, students sometimes confuse the order in which to add/subtract for constant differences, or multiply/divide for constant ratios. Encourage them to identify the pattern, visualize the graph, and make sense of the situation.
- Students often confuse the concepts of percent change and constant multiplier.
- Students struggle with contextual interpretation of the constant multiplier and y-intercept in terms of real-world data.
- It may be difficult for students to understand that a regression model is only a predictor and may not accurately represent what they observe in their own experience.
### Instructional Practices

<table>
<thead>
<tr>
<th>Suggested Timeline</th>
<th>Suggested Learning Experiences</th>
<th>Resources <strong>Common SBG Evidence Items</strong></th>
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<tbody>
<tr>
<td><strong>Exponential Recursion (Week 23)</strong></td>
<td>Explore the differences between arithmetic and geometric sequences. Use different representations (tables, graphs, etc.) to show and continue geometric sequences and make predictions.</td>
<td><strong>Discovering Algebra</strong>: pg. 9 (Investigation); pg. 15 (Example); pg. 333 (Investigation); pg. 338 (6, 8); pg. 339 (9)</td>
</tr>
</tbody>
</table>
| **Exponential Functions (Week 24-25)** | Explicitly write exponential functions that represent specific patterns. Explore the connection between the constant multiplier and the percent rate change. Interpret the meaning of the individual parts of exponential functions in a real-world context. | **Discovering Algebra**: pg. 344 (Example B); pg. 346 (9); pg. 347 (10, 12); pg. 353 (9, 12); pg. 363 (4) | **Illustrative Math**: [Lake Algae](#), [Comparing Exponentials](#), [Basketball Rebounds, A Valuable Quarter](#) | **Dan Meyer’s Three-Act Math**: [Fry’s Bank](#), [Domino Skyscraper](#) | **Mathematics Common Core Toolbox**: [Cellular Growth](#) | **Additional District Created Items**: [Investment Assessment](#) | **Continues on next page**
### Exponential Models (Weeks 26-27)

<table>
<thead>
<tr>
<th>Activity</th>
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<tbody>
<tr>
<td>Fit an exponential curve to real-world data, and create an exponential model that represents the curve.</td>
<td><strong>Discovering Algebra:</strong> pg. 373 (Investigation); pg. 377 (5); pg. 378 (8); pg. 379 (9, 11); pg. 381 (Activity Day)</td>
</tr>
<tr>
<td>Use exponential models to make predictions and decisions. Assess the fit of the models.</td>
<td><strong>Illustrative Math:</strong> [Illegal Fish]**</td>
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<td><strong>PBS Mathline:</strong> <a href="https://pbs.org/mathline/">Exponential Models: Rhinos and M&amp;Ms</a> (use modified assessment African Black Rhino Population**)</td>
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<td><strong>Yummy Math:</strong> [Phone Opening Weekend Sales]**</td>
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<td><strong>Mathalicious:</strong> [Xbox Exponential]**</td>
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### Comparing Linear and Exponential Models (Week 28-30)

<table>
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<tbody>
<tr>
<td>Distinguish between situations that can be modeled with linear functions and with exponential functions. Include sequences, tables, graphs, and real-world situations. Explore end behavior of each type of function.</td>
<td><strong>Discovering Algebra:</strong> pg. 334 (Example A); pg. 339 (11)</td>
</tr>
<tr>
<td>Prove that linear functions grow by equal differences over equal intervals, and exponential functions grow by equal factors over equal intervals.</td>
<td><strong>Illustrative Math:</strong> <a href="https://www.illustrativemathematics.org">Choosing an Appropriate Growth Model, Linear or Exponential?</a> , <a href="https://www.illustrativemathematics.org">Equal Differences over Equal Intervals 2</a> , <a href="https://www.illustrativemathematics.org">Equal Factors over Equal Intervals</a> , <a href="https://www.illustrativemathematics.org">Finding Linear and Exponential Models</a> , <a href="https://www.illustrativemathematics.org">Identifying Functions, Population and Food Supply</a> , <a href="https://www.illustrativemathematics.org">Interesting Interest Rates</a> , <a href="https://www.illustrativemathematics.org">Sandia Aerial Tram</a> , <a href="https://www.illustrativemathematics.org">Exponential Growth versus Linear Growth 1**</a> , <a href="https://www.illustrativemathematics.org">Exponential Growth versus Linear Growth 2</a></td>
</tr>
<tr>
<td>Construct and compare both a linear and exponential model to represent the same situation and solve problems. Have students collect real-life data, when appropriate.</td>
<td><strong>Mathematics Assessment Resource Service:</strong> <a href="https://www.mars.org">Representing Linear and Exponential Growth</a>, <a href="https://www.mars.org">Representing Functions of Everyday Situations</a></td>
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**Differentiation**

- Using a variety of representations can help make functions more understandable to a wider range of students than can be accomplished by working with symbolic representations alone.
- Vary the difficulty of the modeling problems. Begin with data over equal intervals. As students become proficient, use data over unequal intervals.
- Have students use a graphing calculator to compare linear and exponential models.
- Consider the context when using data. Students who struggle with the models should have more familiar context to explore relationships.

**Literacy Connections**

- **Academic Vocabulary**
- **Vocabulary Strategies**
- **Literacy Strategies**

**Additional Resources**

- **Desmos Online Graphing Calculator**